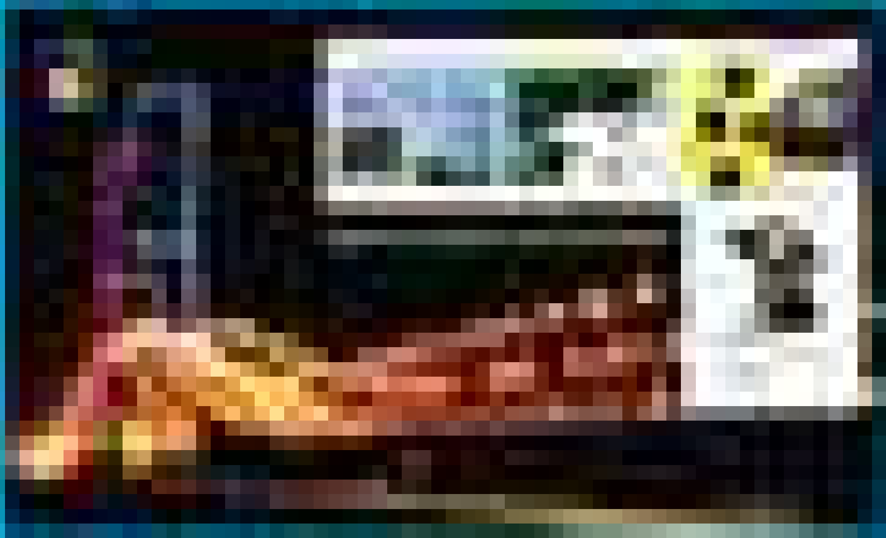

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Investigation of Flood Routing by a Dynamic Wave Model in Trapezoidal Channels

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Abstract. The problems of flood wave propagation, in bodies of waters, cause by intense rains or breaking of control structures, represent a great challenge in the mathematical modeling processes. This research concerns about the development and application of a mathematical model based on the Saint Venant's equations, to study the behavior of the propagation of a flood wave in trapezoidal channels. In these equations, the momentum equation transforms to partial differential equation which has two parameters related to cross-sectional area and discharge of the channel. These new formulas have been solved by using an explicit finite difference scheme. In computation procedure, after computing the discharge from the momentum equation, the cross-sectional area will be obtained from the continuity equation for a given point of channel. To evaluate the behavior of the control variables, several scenarios for the main channel as well as for flood waves are considered and different simulations are performed. The simulations demonstrate that for the same bed width, the peak discharge in trapezoidal channel smaller than in rectangular one at a specific distance along the channel length and so, that roughness coefficient and bed slope of the channel play a strong game on the behavior of the flood wave propagation.

INTRODUCTION

The variation of the discharge with time at a point on a stream channel may be determined by consideration of similar data from a point upstream. In this process, called flood routing, as the flood moves downstream through channel reaches its shape is changed by storage in the reach between any two points. Flood routing helps designers in understanding the flood flow characteristics in river flows and its surrounding area and it is important in designing the flood protection measures and proposing effective economical solutions to protect against flood wave behavior in waterways.

For flood routing problems, one-dimensional Saint-venant equations describe the flood mechanics. Because of difficulties in analytical solution to flood routing in a channel section, various approximations to the Saint-Venant equations have been proposed. Stocker proposed his numerical method known as fixed mesh explicit method for solving unsteady flow equations [9]. To avoid sensivity of explicit methods to the finite time interval, Fox used characteristic lines method proposed by Hartree named also as rectangular grid method [13]. The merit of this method is the easy handling of its simulation and computer programming. Preissmann used implicit scheme and Lip-Frog proposed explicit method from the second stage [5]. Abbott proposed an explicit method, in cooperation with researchers at the Delft University of Netherlands [14]. Abbott's four point method based on characteristic lines method ignored the energy line slope and the bed flow slope to slove the unsteady flow equations. The pioneers of the dynamic wave method are by Preissmann, Blatzer and Lai, Dronkers, Amien and Fang [2]. The most effective effort was done by Amien and Fang for stable, quick, and accurate solution of equations using Newton Raphson iterations. However, most of these solutions need a lot of effort and computer time. Therefore, Keskin and Agiralioglu proposed a new form of dynamic wave model that related with the cross-sectional shape of the channel,

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namely rectangular channel and then it was solved by using an explicit finite difference scheme [11]. This new model was more easy to formulate and simple to compute than that one.

In the prediction of flood flows numerically, in addition to governing equations and their numerical scheme, the main factor affecting the results of the simulation is the cross-sectional shape of the channel through which the flood, because it has effect on the size of the peak discharge in area travesed the flood. Cross-sectional shape of the channel on the ground surface both natural and artificial may be represented by a combination four basic geometric elements as rectangular, triangular, trapezoidal, and a semi-circular. Based on proposed by Keskin, in this study will be investigated the behavior of the propagation of a flood wave in trapezoidal channel. Trapezoidal channel section is selected because it is the most widely open channel sections in engineering. Most of the main water conveying lines have the trapezoidal geometry. The most important advantage of trapezoidal sections is their easy of construction. Beside their constructional advantages, they have also the advantageous of high hydraulic efficiency. Therefore, it is not surprising that most of the water carrying and discharging lines have been made of trapezoidal geometry.

Systematic discussion of this research is build momentum equation for trapezoidal channel that obtained from Saint-Venat equations. In this formulation, the momentum equation transforms to a partial differential equation which has two parameters related to cross-sectional area and discharge of the channel. Hereinafter, the obtained momentum equation and the continuity equation are solved by using explicit finite difference scheme in order to estimate the flood routing for a trapezoidal channel. In computation procedure, after computing the discharge from the momentum equation, the cross-sectional area will be obtained from the continuity equation for a given point of the channel.

In this research will be simulated some cases, including form of outflow at vary positions for entire time, comparison between shape of outflow for trapezoidal channel at a distance of 600 meters with form of outflow for rectangular channel which is selected from the literature, behavior of the flood wave in different time for all positions and the last, simulate how is the influence of the roughness coefficient and bed slope on behavior of the wave propagation.

GOVERNING EQUATION

The basic equations that describe one-dimensional unsteady flow in open channel, the Saint-Venant equations, completely define the flood routing with respect to distance along the channel and time. These equations, called the dynamic wave equations, (Cunge et.al., 1980) can be written in the form of continuity equation and a momentum equation as follows, respectively

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\left(\frac{\partial h}{\partial x} - S_o\right) + gAS_f = 0 \tag{2}$$

where, x is the longitudinal distance along the channel, t is the time, A is the cross-setional area of the flow, Q is the discharge, h is the surface level of the water in the channel, S_o is the slope of bottom of the channel, S_f is the friction slope, g is the gravitational acceleration.

Furthermore, by using Eq. (2) will be formed a new momentum equation which has only two parameters related to cross-sectional area and discharge of the channel. For a trapezoidal cross-sectional as shown in figure 1, cross-sectional area can be written as

$$A = (b + zh)h \tag{3}$$

where b is the bottom width of the channel and z is side slopes of the channel.

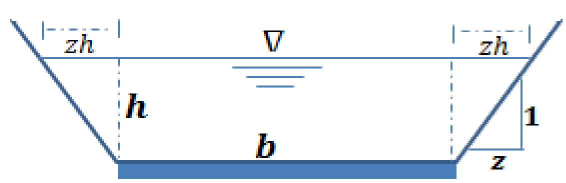

FIGURE 1. Trapezoidal channel cross-section

By assuming the width of channel constant, the following equation can be obtained from Eq. (3) as

$$\frac{\partial h}{\partial x} = \frac{1}{(b+2zh)}\frac{\partial A}{\partial x} \tag{4}$$

By substituting Eq. (4) into Eq. (2) and rearranging the equation one obtains

$$\frac{\partial Q}{\partial t} + 2\frac{Q}{A}\frac{\partial Q}{\partial x} + \left(\frac{gA}{(b+2zh)} - \frac{Q^2}{A^2}\right)\frac{\partial A}{\partial x} + gA\left(S_f - S_o\right) = 0 \tag{5}$$

In order to calculate S_f, the Manning formulation will be used. Thus,

$$V = \frac{1}{n}\, R^{\frac{2}{3}}\, S_f^{\frac{1}{2}} \tag{6}$$

where V is the mean velocity, R is the hydraulic radius, and n is the roughness coefficient. For a trapezoidal channel, the following relationships are given by

$$R = A/P \tag{7}$$

$$P = b + 2h\sqrt{1+z^2} \tag{8}$$

Given the above relations, the following partial derivatives may be listed

$$\frac{\partial P}{\partial x} = 2\sqrt{1+z^2}\,\frac{\partial h}{\partial x} \tag{9}$$

$$\frac{\partial R}{\partial x} = \frac{1}{P}\left(1 - \frac{2\sqrt{1+z^2}}{(b+2zh)}\frac{A}{P}\right)\frac{\partial A}{\partial x} \tag{10}$$

$$\frac{\partial V}{\partial x} = \frac{2}{3}\frac{1}{n}\, R^{-\frac{1}{3}}\, S_f^{\frac{1}{2}}\frac{\partial R}{\partial x} + \frac{1}{2}\frac{1}{n}\, R^{\frac{2}{3}}\, S_f^{-\frac{1}{2}}\frac{\partial S_f}{\partial x} \tag{11}$$

Since $\frac{\partial S_f}{\partial x}$ is very small related to the other terms, the second term in the right side of Eq. (11) can be neglected. Therefore Eq. (11) can be written as

$$\frac{\partial V}{\partial x} = \frac{2}{3}\frac{1}{n}\, R^{-\frac{1}{3}}\, S_f^{\frac{1}{2}}\frac{\partial R}{\partial x} \tag{12}$$

Defining the discharge $Q = VA$, the following partial derivative can be obtained

$$\frac{\partial Q}{\partial x} = A\frac{\partial V}{\partial x} + V\frac{\partial A}{\partial x} \tag{13}$$

By substituting Eqs. (9), (10), and (12) into Eq. (13) and rearranging the equation one obtained

$$\frac{\partial A}{\partial x} = \frac{1}{V\left(\frac{5}{3} - \frac{4}{3}\frac{R\sqrt{1+z^2}}{(b+2zh)}\right)}\frac{\partial Q}{\partial x} \tag{14}$$

And then, by substituting Eq. (14) into Eq. (5), Eq. (15) can be obtained as follows

$$\frac{\partial Q}{\partial t} + \alpha\frac{\partial Q}{\partial x} + \beta = 0 \tag{15}$$

where

$$\alpha = \left(2\frac{Q}{A} + \frac{\frac{gbA}{(b+2zh)} - \frac{Q^2}{A^2}}{\frac{Q}{A}\left(\frac{5}{3} - \frac{4}{3}\frac{R\sqrt{1+z^2}}{(b+2zh)}\right)}\right) \tag{16}$$

$$\beta = gA\left(S_f - S_o\right) \tag{17}$$

The friction slope, S_f can be obtained from the Manning friction formula as $S_f = Q^2 n^2 / \left(A^2 R^{\frac{4}{3}}\right)$ or from Eq. (6).

The momentum equation has two parameters related to cross-sectional area and discharge of the channel. Therefore, Eq. (15) can be solved easily by using a numerical solution subject to initial and boundary conditions.

Initial condition can be written as,

$$Q(x,0) = Q_0 \tag{18}$$

$$A(x,0) = A_0 \tag{19}$$

In which A_0 and Q_0 are the initial values of cross-sectional area and discharge, respectively, for the given inflow hydrograph. The upstream boundary condition can be written as,

$$Q(0,t) = Q_0 + \frac{(Q_p - Q_0)}{t_p}\, t, \quad \text{for } 0 < t < t_p \tag{20}$$

$$Q(0,t) = Q_0 - \frac{(Q_p - Q_0)}{(t_b - t_p)}\, t, \quad \text{for } t_p < t < t_b \tag{21}$$

$$Q(0,t) = Q_0 \tag{22}$$

where, Q_p is the peak flow of inflow hydrograph, t_p is the time to peak flow of inflow hydrograph, t_b is the base time of the inflow hydrograph.

NUMERICAL METHOD

In order to solve the governing equations, an explicit finite difference method is used for numerical solutions. Now, let us consider finite difference approximations for a partial derivative. Let us consider a function $f(x,t)$. We have two independent variables: x and t. We may devide the x-t plane into grid. The grid interval along the x-axis is Δx and the grid interval along t-axis is Δt. We will call the $i\Delta x$ grid point i and the $(i+1)\Delta x$ grid point $i+1$. For the time axis, we will use j for $j\Delta t$ grid point and $j+1$ for $(j+1)\Delta t$ grid point. To refer to different variables at this grid points, we will use the number of the spatial grid as a subscript and that of the time grid as a superscript. We will denote the known time level by superscript j and the unknown time level by $j+1$.

Any dependent variable, $f(x,t)$, and its partial derivatives can be approximated with an explicit finite difference, define as backward in space and forward in time, as follows

$$f(x,t) = f_i^j \tag{23}$$

$$\frac{\partial f(x,t)}{\partial x} = \frac{f_i^j - f_{i-1}^j}{\Delta x} \tag{24}$$

$$\frac{\partial f(x,t)}{\partial t} = \frac{f_i^{j+1} - f_i^j}{\Delta t} \tag{25}$$

Substitution of Eqs. (23) – (25) into momentum equation, Eq. (15), and into continuity equation, Eq. (1), one obtains

$$Q_i^{j+1} = Q_i^j - \frac{\Delta t}{\Delta x}\left(\alpha_i^j\right)\left(Q_i^j - Q_{i-1}^j\right) + \beta_i^j \Delta t \tag{26}$$

$$A_i^{j+1} = A_i^j - \frac{\Delta t}{\Delta x}\left(Q_i^{j+1} - Q_{i-1}^{j+1}\right) \tag{27}$$

Where

$$\alpha_i^j = \left(2\,\frac{Q_i^j}{A_i^j} + \frac{\frac{gbA_i^j}{\left(b^2+2zA_i^j\right)} - \frac{\left(Q_i^j\right)^2}{\left(A_i^j\right)^2}}{\frac{Q_i^j}{A_i^j}\left(\frac{5}{3} - \frac{4}{3}\frac{bR_i^j\sqrt{1+z^2}}{\left(b^2+2zA_i^j\right)}\right)} \right) \tag{28}$$

$$\beta_i^j = gA_i^j\left(\left(S_f\right)_i^j - S_0\right) \tag{29}$$

It can be seen that each pair of α_i^j and β_i^j values can be readily calculated from Eqs. (28) and (29) using the known initial and boundary data at starting point of (i,j) the one can obtain Q_i^{j+1} from Eq. (26). Finally, using Q_i^{j+1}, A_i^{j+1} can be calculated from Eq. (27). This technique will be repeated for successive values of (i,j).

RESULTS

In this numerical simulation, it was considered the channel with trapezoidal section, with the following characteristics: the length of the channel $L = 2000\ m$, the bottom width of the channel $b = 5\ m$, the bottom slope of the channel $S_0 = 0.0005$, and its Manning's roughness coefficient $n = 0.0138$. for the triangular inflow hydrograph, the hydrograph is selected as $Q(x,t) = Q(0,0) = Q_0 = 3\ m^3/s$, $Q(0,10) = Q_p = 12\ m^3/s$, $Q(0,20) = Q_b = 3\ m^3/s$.

For the accuracy of the results, Δx should be small and for the achievement of the stability in practice, $\frac{\Delta t}{\Delta x}$ should be smaller than Courant criteria. The space interval was selected as $\Delta x = 100\ m$ and the time interval was selected as $\Delta t = 10\ s$. Larger values of Δt have been tested, but they given results which deviated from the expected values.

The dynamic wave model developed in this work was to accomplish several simulations, for the same initial and boundary conditions, in way to evaluate the propagation of the wave along the channel, as follows.

Behavior of the flow for different time

The figure 2 represent the behavior of the flow, for different time, showing the wave propagation with the space. Through the figure 2 below, it can be observed that the wave has a regular propagation altering the value of the flow systematically in the time and in the space. The difference, among the distance and corresponding to the positions of two consecutive crests, that is, devide by increment of time, corresponding is the celerity of the wave.

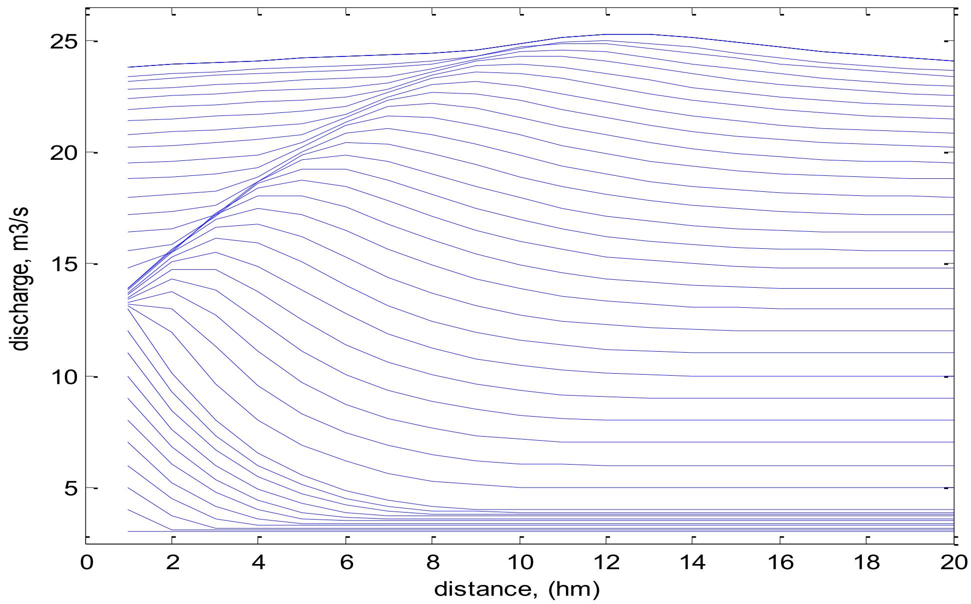

FIGURE 2. Wave propagation for different time

Behavior of the flow for different distance

In the figure 3 the propagation of the wave can be observed, along the time, for different sections of the channel. Through the figure 3, it is noticed that the energy of propagation of the wave vanishes quickly along the time. That is caused mainly by the friction factor present in the equation of Saint Venant, and consequently, this result obtained in the present simulation is entirely in agreement with the theoretical subject of the model.

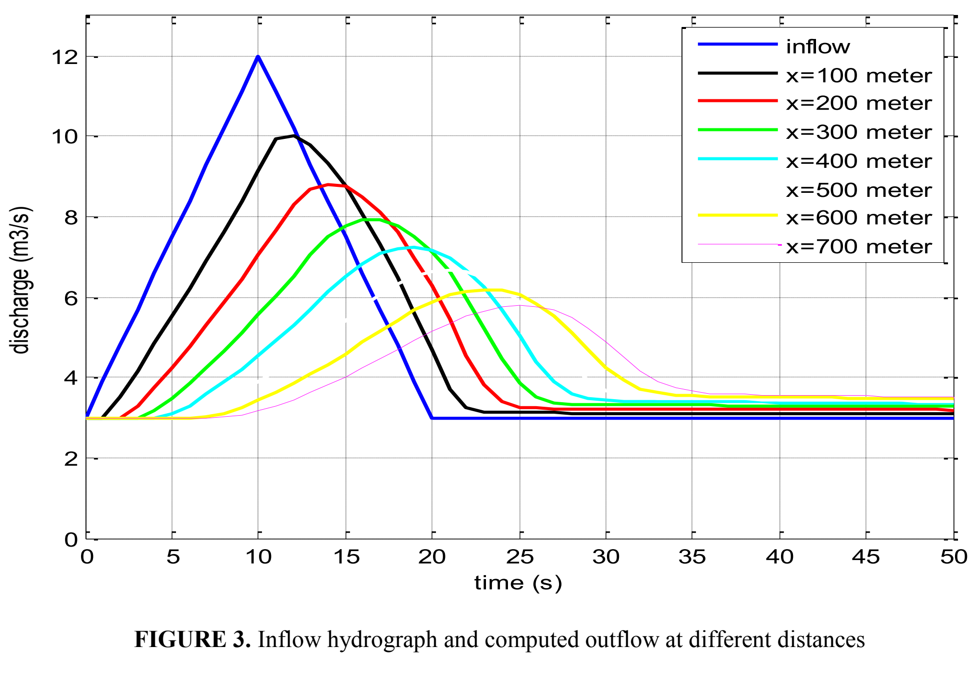

FIGURE 3. Inflow hydrograph and computed outflow at different distances

Comparison outflow in trapezoidal and in rectangular

In order to compare the simplified dynamic model in a trapezoidal channel with in a rectangular one event is selected from literature as used by Keskin.

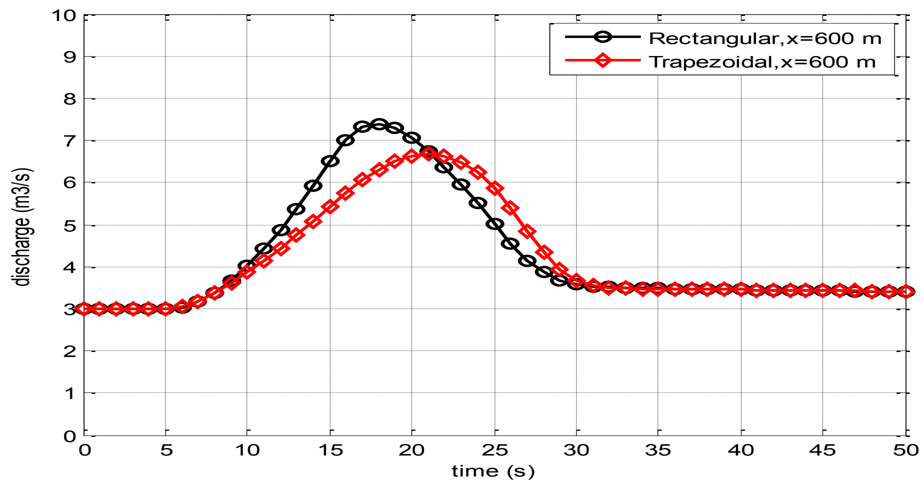

FIGURE 4. Comparison of outflow hydrograph for trapezoidal channel and rectangular one at distance of 600 m

They used the same inflow hydrograph and selected the distance as 600 m. Figure 4 indicate two computed hydrographs, one of them is obtained in trapezoidal channel, the other was found in rectangular one. The maximum discharge of trapezoidal channel is 6.67 m^3/s and its time is 22 min, whereas for rectangular one found a peak discharge of 7.38 m^3/s and a time of 19 min. Through the figure 4 above, it can be observed that the arrival of awave in a fixed point of the channel, showing that as bigger is the free surface (top) width the smaller will be pick of the flood wave.

Behavior of the wave propagation for different bed slope and roughness

The figure 5 shows the propagation of the wave for different bed slope, for the section distant 600 meter from the origin, in function of the time. This simulation shows the arrival of a wave in fixed point of the channel, showing that as bigger is the bed slope the bigger will be the pick of the flood wave. However, the results show that the speed of propagation of the wave does not change so much with this parameter. Through the fgure, it is possible to see that the time that the pick of wave reaches the section 600 m from the origin, is, approximately, of 22 minuts, for all simulation.

The analysis could be done with respect to the influence of the roughness coefficient on the behavior of the wave propagation for section distant 600 meter from origin, as shown in figure 6. This simulation shows the arrival of a wave in a fixed point of the channel, showing that as bigger is the roughness cefficient the smaller will be pick of the flood wave.

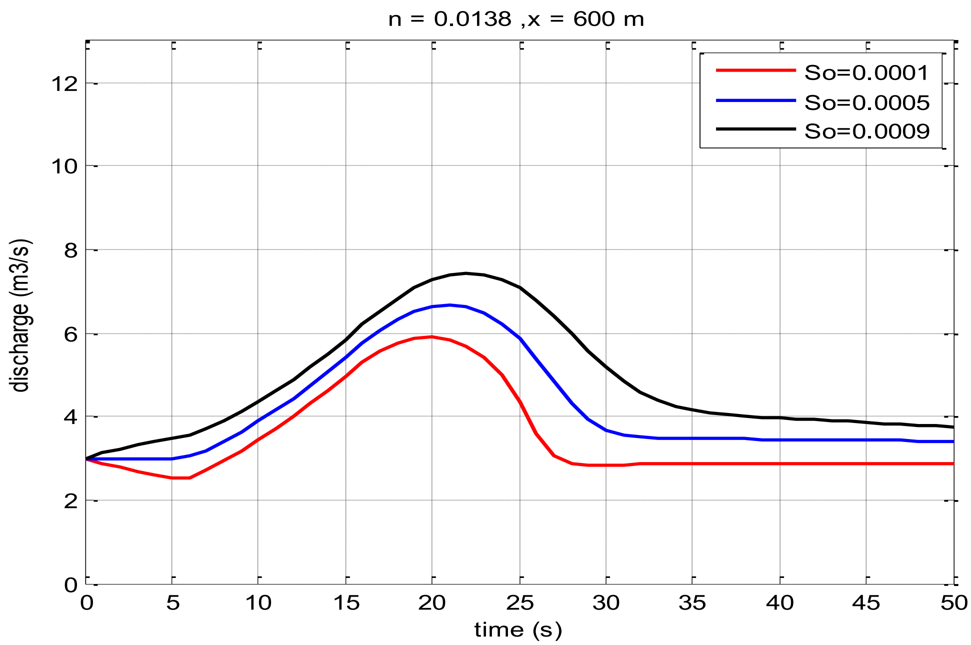

FIGURE 5. Behavior of the wave propagation for different bed slope

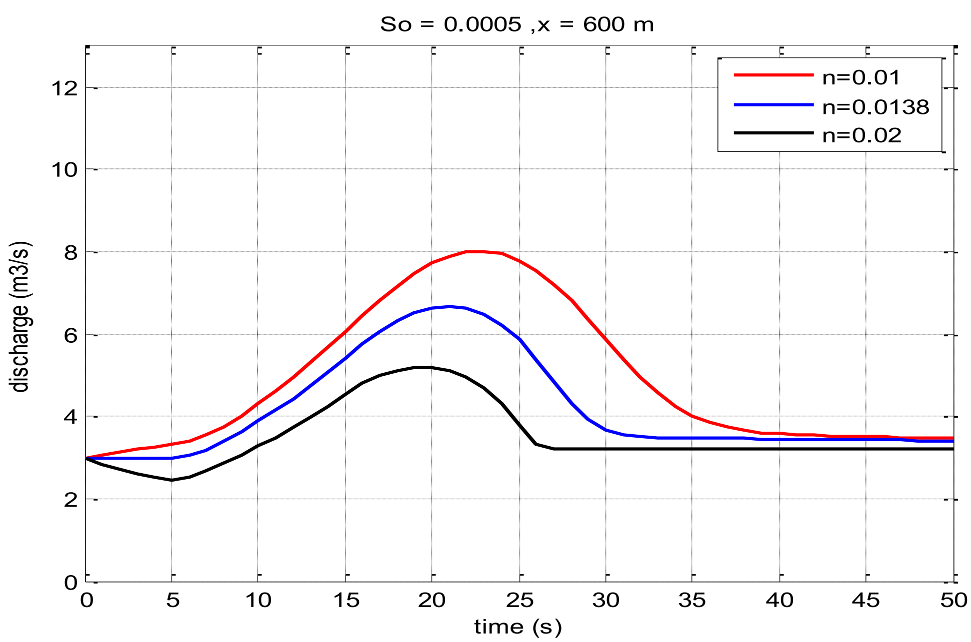

FIGURE 6. Behavior of the wave propagation for different roughness

CONCLUSIONS

For flood routing calculation in trapezoidal channel, a free surface flow model has been developed. The obtained result from dynamic wave model in trapezoidal channel is compared with in rectangular one undar the same conditions. Finding indicate that as bigger is the free surface (top0 width the smaller will be the pick of the flood wave.

And also with respect to yhe behavior of the flood wave, it was verified that, for different bed slope of the channel, the propagation of the wave suffers important influence of this parameter, allowing, so, to conclude that as larger the bed slope is, the bigger will be the pick of the flood wave.

Finally, the results showed that this parameter does not make a great influence on the celerity of the flood wave. It was verified that for a same distance of origin, the picks of the wave arrive, approximately, at the same time.

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