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Numerical Simulation of Flood Routing by the Simplified Saint Venant Equations in Rectangular Channels

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Abstract

Floods, which cause a lot of damage, are a natural phenomenon that often occurs during the rainy season. Flood occurs because the discharge entering the channel exceeds the channel capacity. If the discharge data in the upstream area that will enter the channel is known, then we can determined the flow behavior in the downstream area using a mathematical model. In this study, we proposed to use a simplified Saint Venant equations for simulating the flow routing in a prismatic channel with rectangular section. This model is solved numerically using the finite difference method. Here, the numerical scheme used succeeds in simulating the flow behavior in the channel due to the discharge entering it. The simulation results show that the discharge entering the channel will propagate downstream with the decreasing quantity of discharge. Information on the amount of discharge at locations along the channel is useful as supporting data for flood control and prevention systems that will be conveyed to residents along the channel..
Keywords: flood routing, prismatic channel, Saint Venant Equations, finite difference method.

Abstrak

Banjir yang menimbulkan banyak kerusakan merupakan fenomena alam yang sering terjadi pada musim hujan. Banjir terjadi karena debit yang masuk ke dalam kanal melebihi kapasitas kanalnya. Jika data debit di daerah hulu yang akan masuk ke dalam kanal diketahui, maka kita dapat menentukan perilaku aliran di daerah hilir dengan menggunakan model matematika. Dalam studi ini, kami mengusulkan untuk menggunakan persamaan Saint Venant yang disederhanakan untuk mensimulasikan penelusuran aliran pada saluran prismatik dengan penampang persegi panjang. Model ini diselesaikan secara numerik dengan menggunakan metode beda hingga. Di sini, skema numerik yang digunakan berhasil mensimulasikan perilaku aliran pada saluran akibat debit yang masuk. Hasil simulasi menunjukkan bahwa debit yang masuk ke saluran akan merambat ke hilir dengan kuantitas debit yang semakin berkurang. Informasi jumlah debit di lokasi sepanjang saluran ini berguna sebagai data pendukung pada sistem pengendalian dan pencegahan banjir yang akan disampaikan kepada penduduk di sepanjang kanal.
Kata kunci: penelusuran banjir, saluran prismatik, persamaan Saint Venant, metode beda hingga.

1. INTRODUCTION

Floods can cause damage to agricultural areas, industrial areas and cities. Floods also cause disruption of human activities, damage to facilities and infrastructure, loss of property and even human lives. In general, flooding occurs because the channel is unable to accommodate the discharge entering the channel. To prevent damage from flooding, water structures such as embankments, weirs, etc. are usually constructed at several locations along the channel. To design a water structure, information is always required about the amount of discharge at the location where the water structure will be erected. Information on the amount of discharge is important to determine the strength of the water structure to be built so that it does not break.

There are many ways or methods that can be used to determine the amount of discharge at the required locations along the canal. In this study, we propose the use of a mathematical model that can describe the flow behavior in the channel in the form of the Saint Venant equation. The Saint Venant equation is composed of two hyperbolic type partial differential equations, namely the continuity equation and the momentum equation. This model is a nonlinear system of equations and the analytical solution has not been found. Therefore, in practice, some researchers simplify the Saint Venant equation by way of ignoring part of the terms from the momentum equation to be excluded from the modeling. Thus, various research models emerged, including the kinematic model [1], the diffusion model [2], and the Keskin model [3], [4]. Meanwhile, other researchers developed certain numerical methods for solving Saint Venant's equations without simplification. Among the numerical methods used include the characteristic method [5], [6], the finite difference method [7], [8], the finite element method [9], and the finite volume method [10], the finite volume method on the grid staggred [11], [12].

In this research, the simulation of flow routing in the prismatic channel was carried out using the Saint Venant equations with a simplification called the Keskin model [3]. Unlike other simplification models, the simulation results from the Keskin model have a high compatibility with the simulation results from the model without simplification, do not require a lot of computation time and can be solved by relatively simple numerical methods.

2. METHOD

2.1 Governing Equation

The mathematical model that can be used to describe the flow behavior in a channel is the Saint Venant equations [5]. This equation is composed of two simultaneous equations, namely the continuity equation and the momentum equation. The Saint Venant equations can be written as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial h}{\partial x} + gA\left(S_f - S_0\right) = 0 \tag{2}$$

where A is the wet cross-sectional area, Q is the discharge, h is the water level, S_f is the slope of friction, S_0 is the slope of bottom channel, g is the acceleration due to gravity, t is coordinate of time, and x is the coordinate of distance.

For flood routing problems in a prismatic channel (it is assumed that the channel width b is constant) with a rectangular section, the wet cross-sectional area A can be written as

$$A = bh \tag{3}$$

Equation (3) is derived with respect to x, then we obtain

$$\frac{\partial A}{\partial x} = b\frac{\partial h}{\partial x} \tag{4}$$

Substitute equation (4) into equation (2) so we obtain

$$\frac{\partial Q}{\partial t} + 2\frac{Q}{A}\frac{\partial Q}{\partial x} + \left(\frac{gA}{b} - \frac{Q^2}{A^2}\right)\frac{\partial A}{\partial x} + gA\left(S_f - S_0\right) = 0 \tag{5}$$

When the flow occurs in an open channel, due to the roughness of the walls and bottom of the channel as a barrier where the flow is located, there will be a resistance flow from upstream to downstream which will have a direct effect on the magnitude of the velocity. Calculation of flow velocity through an open channel can only be done using empirical formulas, because there are many variables that change. In this regard, experts such as Chezy, Manning or Strickler provide empirical formulas which are built on the basis of steady uniform flow formula. If the Manning formula is selected, then the formula for expressing the resistance to flow can be written as

$$S_f = \frac{n^2 u^2}{R^{4/3}} \tag{6}$$

where n denotes the Manning coefficient, R denotes the hydraulic radius, and u denote the velocity.

For a channel with rectangular cross section, the following relationships apply

$$R = \frac{A}{P} \tag{7}$$

$$P = 2h + b \tag{8}$$

with P denotes the wet perimeter of the cross section. Furthermore, equations (6), (7), and (8) are derived partially with respect to x, obtained

$$\frac{\partial P}{\partial x} = 2\frac{\partial h}{\partial x} \tag{9}$$

$$\frac{\partial R}{\partial x} = \frac{1}{P}\left(1 - \frac{2A}{bP}\right)\frac{\partial A}{\partial x} \tag{10}$$

$$\frac{\partial u}{\partial x} = \frac{2}{3}\frac{1}{n}R^{-\frac{1}{3}}S_f^{\frac{1}{2}}\frac{\partial R}{\partial x} + \frac{1}{2}\frac{1}{n}R^{\frac{2}{3}}S_f^{-\frac{1}{3}}\frac{\partial S_f}{\partial x} \tag{11}$$

According to [3], the value of sf is very small when compared to the values of other terms, so that the second term in the right side in equation (11) can be ignored. Therefore, Equation (11) can be rewritten as

$$\frac{\partial u}{\partial x} = \frac{2}{3}\frac{1}{n} R^{-\frac{1}{3}} S_f^{\frac{1}{2}} \frac{\partial R}{\partial x} \tag{12}$$

In a steady flow, the discharge is defined as $Q = Au$. If it is derived partially with respect to x, it is obtained

$$\frac{\partial Q}{\partial x} = A\frac{\partial u}{\partial x} + u\frac{\partial A}{\partial x} \tag{13}$$

Then substitute equations (9), (10), and (12) into equation (13) and be rearranged of the equation, we obtain

$$\frac{\partial A}{\partial x} = \frac{1}{\frac{Q}{A}\left(\frac{5}{3}-\frac{4R}{3b}\right)}\frac{\partial Q}{\partial x} \tag{14}$$

Next, substitute equation (14) into equation (5) and be rearranged of the equation, we get

$$\frac{\partial Q}{\partial t} + \alpha\frac{\partial Q}{\partial x} + \beta = 0 \tag{15}$$

where

$$\alpha = \left(2\frac{Q}{A} + \frac{\frac{gA}{b}-\frac{Q^2}{A^2}}{\frac{Q}{A}\left(\frac{5}{3}-\frac{4R}{3b}\right)}\right) \tag{16}$$

and

$$\beta = gA\left(S_f - S_0\right) \tag{17}$$

The momentum equation has two variables related to the cross-sectional area and the discharge of the channel. Therefore Equations (1) and (15) can be solved using different numerical methods until they are in accordance with the initial conditions and the boundary conditions.

2.2 Numerical Method

The Saint Venant equations (1) and (15) will be solved numerically using the finite difference method. With this method, the equation will be converted into a different equation that only applies to certain points in the domain of solution. Since these equations contain the independent variables x and t, an approximation of the difference equation to it is done by creating a number of grid on the $x - t$ plane.

If the partial differential equation has an exact solution $f(x,t)$, then the difference equation will have an approximate solution of $f(x_i, t^k)$. Here, any function $f(x,t)$ and its partial derivatives will be discretized by an explicitly different method (forward time and backward space) using the following rules.

$$f(x,t) \approx f(x_i, t^k) = f_i^k \tag{18}$$

$$\frac{\partial f(x,t)}{\partial t} \approx \left.\frac{\partial f}{\partial t}\right|_i^k = \frac{f_i^{k+1} - f_i^k}{\Delta t} \tag{19}$$

$$\frac{\partial f(x,t)}{\partial x} \approx \left.\frac{\partial f}{\partial x}\right|_i^k = \frac{f_i^k - f_{i-1}^k}{\Delta x} \tag{20}$$

where x and t are the length of the interval in the direction x and the length of the interval in the direction t, respectively. Substitute equations (18), (19), (20) into the momentum equation (15) and the continuity equation (1), we get

$$Q_i^{k+1} = Q_i^k - \frac{\Delta t}{\Delta x}\left(\alpha_i^k\right)\left(Q_i^k - Q_{i-1}^k\right) - \beta_i^k \Delta t \tag{21}$$

$$A_i^{k+1} = A_i^k - \frac{\Delta t}{\Delta x}\left(Q_i^{k+1} - Q_{i-1}^{k+1}\right) \tag{22}$$

where

$$\alpha_i^k = 2\frac{Q_i^k}{A_i^k} + \frac{\frac{gA_i^k}{b} - \left(\frac{Q_i^k}{A_i^k}\right)^2}{\frac{Q_i^k}{A_i^k}\left(\frac{5}{3} - \frac{4}{3}\frac{R_i^k}{b}\right)} \tag{23}$$

$$\beta_i^k = gA_i^k\left(S_{f_i}^{\ k} - S_0\right) \tag{24}$$

From the explanation above, it can be seen that each pair of values of α_i^k and β_i^k can be calculated from Equations (23) and (24) using the initial condition data and the boundary conditions that are known at the beginning (i, k) so that the value Q_i^{k+1} can be obtained from Equation (21). Finally, using the value Q_i^{k+1}, the value A_i^{k+1} can be calculated from Equation (22). This technique will be repeated sequentially against the values of (i, k).

3. RESULT AND DISCUSSION

The numerical schemes from equations (21) and (22) will be applied to the problem of flow routing in a prismatic channel. A prismatic channel with a rectangular cross section is fed by water from the upstream direction. By utilizing the discharge data that enters the prismatic channel, in the downstream area will observe changes in the behavior of two water flow variables, namely discharge and water level elevation. The general components used for numerical simulations are as follows: channel length $L = 2000$ m, constant channel width $b = 5$ m, channel base slope $S_f = 0.0005$, Manning roughness coefficient $n = 0.0138$, gravitational acceleration $g = 9{,}81\ m/s^2$, space step $\Delta x = 1$ m, time step $\Delta t = 0.1$ second and the simulation is carried out for 60 units of time, $T = 60\ s$.

The next presentation is to carry out a simulation of tracing water flow in prismatic channel due to variations in the incoming hydrograph inflow, namely the triangular hydrograph. Then investigate the behavior of water flow variables will be observed in the form of discharge and water level at certain locations in the downstream area.

3.1 Hydrograph Inflow

A prismatic channel with a rectangular cross section is fed by water from the upstream direction. From the upstream direction, a triangular-shaped hydrograph inflow is recorded into the channel with the following data,

$$Q(0,t) = \begin{cases} 3 + \frac{9}{10}t & \text{jika} \quad 0 \le t < 10 \\ 12 - \frac{9}{10}(t-10) & \text{jika} \quad 10 \le t < 20 \\ 3 & \text{jika} \quad t \ge 20 \end{cases} \tag{25}$$

where the initial condition is $Q\,(x,0) = 3\ m^3/s$, $A\,(x,0) = 3\ m^3/s$. The hydrograph inflow profile that enters the prismatic channel can be seen in Figure 1.

The figure 1 shows the quantity that discharge that will enter the prismatic channel gradually over 20 s, $0 < t \le 20$ s. Initially, the discharge entering the prismatic channel is quite small, which is $Q = 3\ m^3/s$. As time increases, the incoming debit to fill the channel gets bigger until it reaches its peak when $t = 10$ s, which is $Q = 12\ m^3/s$. After that, the incoming debit gradually decreases until no more debit enters the channel and this happens when $t = 20$ s. Based on upstream discharge data like this, in the next discussion, several simulation results will be displayed regarding changes in the behavior of water flow variables in the form of discharge (outflow hydrograph) and water level elevation in the downstream area.

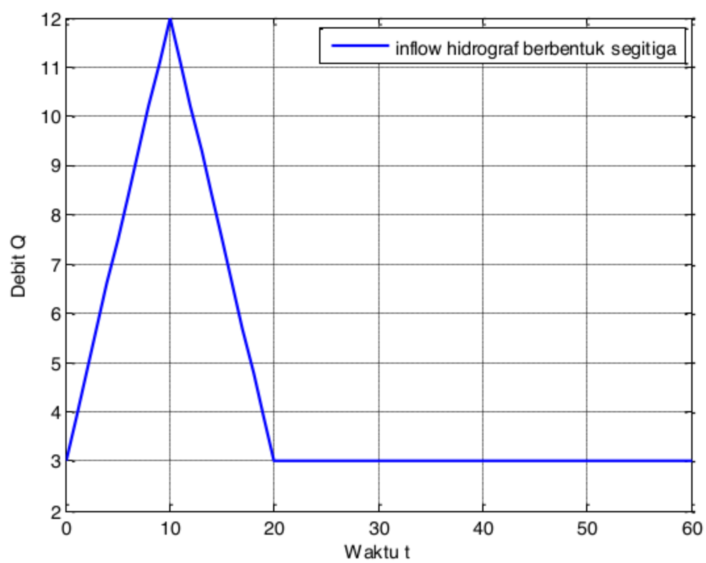

Figure 1. The hydrograph inflow profile is triangular in shape that enters a prismatic channel.

3.2 Hydrograph Outflow

The simulation results of changes in water flow variables in the form of hydrograph outflow along the canal can be seen in Figure 2. The figure shows an animation of downstream flow movements for a time of $0 < t \le 60$ seconds due to the triangular hydrograph inflow that enters the channel for a time of $0 < t \le 20$ seconds.

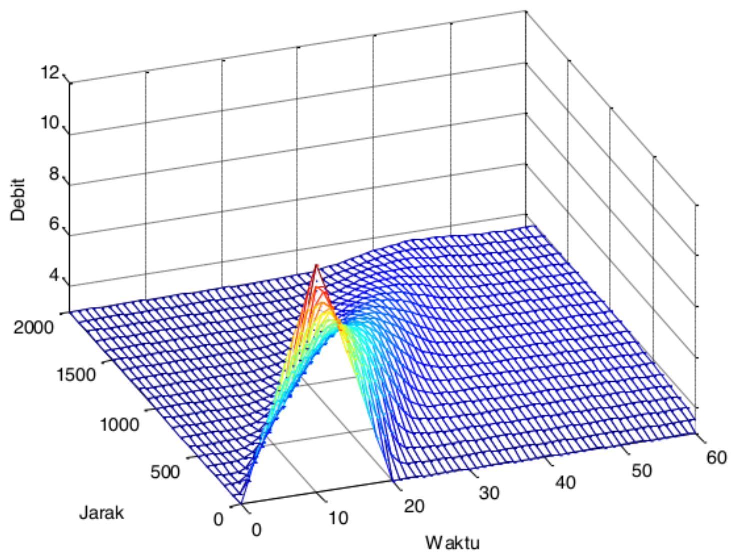

Figure 2. Discharge profile due to a triangular hydrograph inflow.

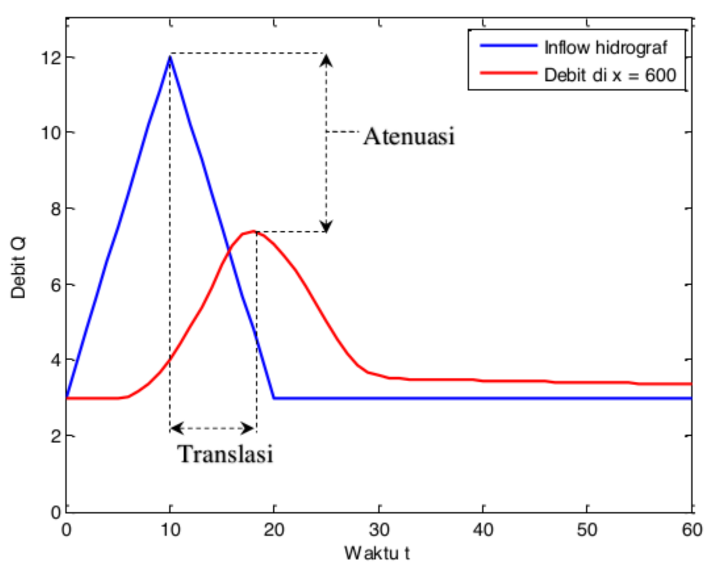

Figure 3. Comparison between triangular hydrograph inflow and discharge at x = 600 m which indicates a decrease in peak discharge.

From the figure, it can be seen that the discharge that enters the channel will propagate downstream for each unit of time with the quantity of discharge that is getting smaller and smaller until the effect of the hydrograph inflow is gone. The hydrograph inflow will experience damping along the canal due to the influence of friction between the water and the bottom and walls of the canal so that the farther from the source of discharge, the greater the shrinkage of the discharge.

In Figure 3 is presented a triangular hydrograph inflow and a hydrograph outflow observed at a location 600 m from the upstream. From the figure, it can be seen that the maximum discharge has decreased from $Q = 12\ m^3/s$ to $Q = 7.38\ m^3/s$ and the time of the maximum discharge has also shifted from $t = 10$ s to $t = 18$ s.

4. CONCLUSION

The Saint Venant equation is a mathematical model that can be used to describe the phenomenon of water flow in a channel. The model is built based on two natural laws, namely the law of conservation of mass and the law of conservation of momentum. These two laws are commonly expressed in mathematical equations known as the continuity equation and the momentum equation.

Based on the observation of the numerical simulation results in this research, it can be concluded that the hydrograph inflow that enters the prismatic channel will experience shrinkage along the channel due to friction between the water flow and the bottom and walls of the channel. The farther from the discharge source, the greater the shrinkage of the water flow variable. For observations at a certain location, the size of the variable quantity of water flow depends on the size of the incoming hydrograph inflow quantity.

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