Bambang Agus Sulistyono paper 2

by Prodi Matematika

Submission date: 20-Dec-2022 01:55AM (UTC-0500)
Submission ID: 1984914531
File name: Advances_and_Applications_in_Fluid_Mechanics.pdf (545.69K)
Word count: 2036
Character count: 9933

Advances and Applications in Fluid Mechanics

http://www.pphmj.com
http://dx.doi.org/10.17654/FM023020171
Volume 23, Number 2, 2019, Pages 171-179 **ISSN: 0973-4686**

A STAGGERED METHOD FOR NUMERICAL FLOOD ROUTING IN RECTANGULAR CHANNELS

B. A. Sulistyono and L. H. Wiryanto

Department of Mathematics
Bandung Institute of Technology
Jalan Ganesha 10 Bandung, Indonesia

Abstract

We consider the Saint Venant's equations applied to flood routing in rectangular open channels. The model is solved numerically using a finite volume method on staggered grids. We select the use of this method, because Riemann solutions are not needed so that the numerical calculation does not require a lot of efforts and the numerical computation is cheap to do. Simulation results indicate that our proposed method is successful in solving the problems. In addition, we obtain that the greater value of the friction leads to the smaller value of the discharge.

1. Introduction

Flooding is a natural phenomenon that generally occurs due to the inability of the channel to contain incoming water. If the water that enters the channel exceeds the channel capacity, then it will cause flooding that has an impact on the destruction of facilities and infrastructure, disruption of human activities, and loss of property and even human lives.

Received: September 23, 2019; Accepted: October 24, 2019

2010 Mathematics Subject Classification: 76-XX.

Keywords and phrases: Saint Venant's equations, flood routing, finite volume method.

The Saint Venant's equations as a hydraulic model have been widely applied to the problem of flood routing in river, see for examples [1-3, 6, 10, 11]. These equations are highly non-linear and there do not have analytical solution. Therefore, several numerical methods such as characteristic methods [1], finite difference methods [1-3, 6, 8], finite element methods [11], and finite volume methods [9] have been used to solve this Saint Venant's equations. However, most of these solutions require a lot of attempts and numerical computation time.

In implementing of finite volume method, the velocity and the water depth variables of the Saint Venant's equations are approximated on the same grid. This matter causes Riemann solutions are required in order to approximate the numerical fluxes. If the equations are approximated on the staggered grid, then the numerical fluxes can be computed by component using the upwind approximation. Therefore, Riemann solutions are not needed. This is the great advantage of using the staggered-grid method, so that it makes our numerical technique more simple and the numerical computation is not expensive to do. In this study, we propose to develop a finite volume method on staggered grid in order to solve the Saint Venant's equations applied to flood routing in rectangular open channels. Furthermore, to investigate outflow hydrographs (discharge) for different distance and different friction' factor.

The paper is organized as follows. The Saint Venant's equations applied to rectangular open channels are presented in Section 2. The proposed numerical method is presented in Section 3. Numerical results are presented in Section 4. Some concluding remarks are presented in Section 5.

2. Mathematical Model

The basic equation commonly used to describe the behavior of water flow in an open channel is the Saint Venant's equations. This system is represented by continuity equation and momentum equation [1] that can be written as follows, respectively

$$A_t + Q_x = 0, \tag{1}$$

$$Q_t + \left(\frac{Q^2}{A}\right)_x + gAh_x + gA(S_f - S_0) = 0, \tag{2}$$

where A states the wet cross-sectional area, Q is the discharge, h denotes the water depth, g is the gravitational acceleration, S_0 is the slope of the base of the channel, and S_f denotes the slope of the friction.

For flood routing in rectangular open channels, cross-sectional area and discharge can be formulated as

$$A = b(x)h(x, t), \tag{3}$$

$$Q = b(x)h(x, t)u(x, t). \tag{4}$$

Substituting equations (3) and (4) into equations (1) and (2), we get

$$h_t + (hu)_x = -\frac{hub_x}{b}, \tag{5}$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -\frac{hu^2b_x}{b} - ghS_0 - ghS_f. \tag{6}$$

By substituting $h_t = -(hu)_x - \frac{hub_x}{b}$ into equation (6) and denote $q \equiv hu$, then we get the equivalent momentum equation, i.e.,

$$u_t + \frac{1}{h}(qu)_x + \frac{u}{h}(q)_x + \frac{1}{h}\left(\frac{1}{2}gh^2\right)_x + gS_0 + gS_f = 0, \tag{7}$$

where u is the velocity, $b = b(x)$ represents the channel width.

In this paper, equations (5) and (7) are the mathematical model which shall be solved numerically using the finite volume method on staggered grid.

3. Numerical Method

We consider the Saint Venant's equations (5) and (7) on the spatial domain $0 \le x \le L$ and the interval of time $0 \le t \le T$. The interval of time is divided into N_t time steps with length Δt, for all $k \in \{0, ..., N_t\}$, $t^k = k\Delta t$. The spatial domain is partitioned into N_x cells with length Δx. The left end, the center and the right end of the i-th cell are symbolized by $x_{i-1/2}$, x_i and $x_{i+1/2}$, respectively. Here, the velocity u is partitioned at the interfaces between the cells, whereas the height of water h and the width of channel b are partitioned at the center of the cells (see Figure 1). This picture illustrates both the continuity cell $[x_{i-1/2}, x_{i+1/2}]$ centered at the grid point x_i, where the continuity equation (5) is estimated, and the adjacent cell $[x_i, x_{i+1}]$ centered at the staggered grid point $x_{i+1/2}$, where the momentum equation (7) is estimated. The estimation of u at point $x_{i+1/2}$ and at time t^k is symbolized by $u^k_{i+1/2}$. The estimation of h at point x_i and at time t^k is symbolized by h^k_i. The estimation of b at point x_i is symbolized by b_i.

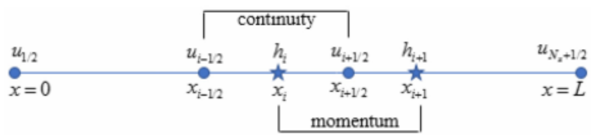

Figure 1. Illustration of a one-dimensional finite volume scheme on staggered grid.

The discretization of the continuity equation (5) is given by

$$h_i^{k+1} = h_i^k + \frac{\Delta t}{\Delta x}[q^k_{i+1/2} - q^k_{i-1/2}] + \frac{\Delta t}{\Delta x}\left[h_i^k {}^* u_i^k \frac{b_{i+1/2} - b_{i-1/2}}{b_i}\right], \tag{8}$$

where

$$q^k_{i+1/2} = {}^*h^k_{i+1/2} u^k_{i+1/2}, \tag{9}$$

$${}^*h^k_{i+1/2} = \begin{cases} h_i^k & \text{if } u^k_{i+1/2} \geq 0, \\ h^k_{i+1} & \text{if } u^k_{i+1/2} < 0, \end{cases} \tag{10}$$

$$^*u_i^k = \begin{cases} u_{i-1/2}^k & \text{if } q_i^k \geq 0, \\ u_{i-1/2}^k & \text{if } q_i^k < 0. \end{cases} \tag{11}$$

The discretization of the momentum equation (7) is given by

$$u_{i+1/2}^{k+1} = u_{i+1/2}^k - \frac{\Delta t}{h_{i+1/2}^{k+1}\Delta x}(q_{i+1}^k\, {}^*u_{i+1}^k - q_i^k\, {}^*u_i^k)$$

$$+ \frac{\Delta t}{h_{i+1/2}^{k+1}\Delta x}[u_{i+1/2}^k(q_{i+1}^k - q_i^k)]$$

$$- g\frac{\Delta t}{\Delta x}[(h_{i+1}^{k+1} - h_i^{k+1}) + S_0] - g\Delta t S_f, \tag{12}$$

where

$$h_{i+1/2}^k = \frac{1}{2}(h_{i+1}^k + h_i^k), \tag{13}$$

$$q_i^k = \frac{1}{2}(q_{i+1/2}^k - q_{i-1/2}^k). \tag{14}$$

In the schemes (8) and (12) above, we do not apply any Riemann solver in the calculation of numerical flux, so that the numerical computation is more simply to do.

4. Results

The common characteristics for simulating are selected as follows: the spatial domain is $[0, 2000]$, the channel width is $b = 5\,\text{m}$, the channel bottom slope is $S_0 = 0.0005$, $S_f = 0.0138$ and the boundary conditions is $h(0, t) = 0.75$ and $u(0, t) = 0.8$. For inflow hydrograph is selected as

$$q(0, t) = \begin{cases} 3 + \frac{9}{10}t & \text{if } 0 \leq t < 10 \\ 12 - \frac{9}{10}(t - 10) & \text{if } 10 \leq t < 20 \\ 3 & \text{if } t \geq 20. \end{cases} \tag{15}$$

Here, the number of the spatial cells is 800 in which a half of them are used to compute the water depth and the others are used to compute the velocity.

4.1. Comparison with simplified dynamic model

In this section, we compared the results of the simulation of a staggered scheme with article written by [2]. Simulation results of the proposed model and the Keskin model can be seen in Figure 2. The figure shows that the increase in hydrograph of the proposed model is the same as the Keskin model. In contrast, the recession hydrograph of the proposed model decreases slightly more slowly than the Keskin model. The arrival of the proposed model in the base stream is the same as the Keskin model. An investigation of this curves shows that there are no significant differences between the models.

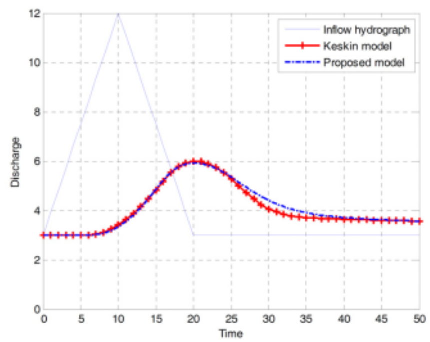

Figure 2. Comparison of discharge for the proposed model and simplified dynamic model at distance of 600 m.

4.2. Outflow hydrograph with different distance

The computed discharges are shown in Figure 3 for different distance, such as $x = 400, 500, 600$ m. As seen from this figure that discharges have crest flows of 6.7758, 6.4128, 6.0678 m^3/s. The times to crest flows are 13, 16, 19 min, respectively. From Figure 3 looked at that the propagation of energy wave dissolve quickly along the time. That is mainly due to the friction's factor in the Saint Venant's equations.

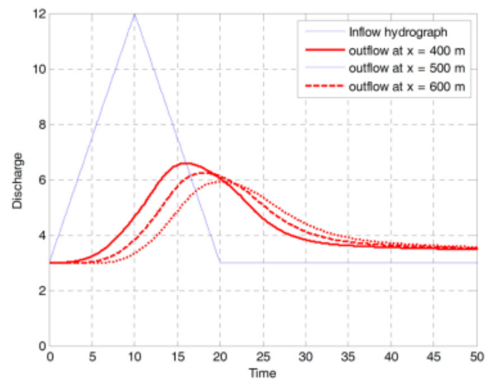

Figure 3. Calculated discharges at $x = 400{,}500{,}600$.

4.3. Outflow hydrograph with different friction factor

The computed discharges are shown in Figure 4 for different friction's factor, such as $S_f = 0.01, 0.09, 0.15$. As seen from this figure that discharges have crest flows of 8.94, 8.28, 7.30 m^3/s, respectively. The times to crest flows are 12, 13, 14 min, respectively. Figure 4 shows that the greater the value of the friction's factor, the smaller the value of the discharge. This is due to the friction's factor is an important factor in the resistance of flow.

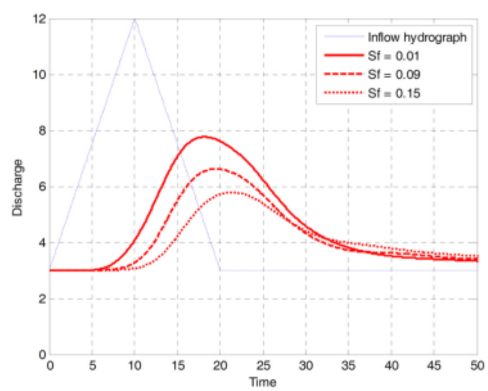

Figure 4. Calculated discharges for different friction's factor.

5. Conclusion

A finite volume scheme on staggered grids has been developed for solving Saint Venant equations applied to flood routing in prismatic open channel with rectangular cross-section. The advantage of using this method is that the numerical flux can be calculated more simply because the requirement of the use of the Riemann solver can be avoided, so that the numerical computation is not expensive to do. Simulation results have shown that the hydraulic parameters play an important game in the flood mechanics.

Acknowledgement

The research of the problem was supported by Research Gant from Indonesian Government, year 2019. Therefore, the authors gratefully acknowledged to RISTEK-DIKTI.

References

[1] J. A. Cunge, F. M. Holly, Jr. and A. Verwey, Practical aspects of computational river hydraulics, Pitman Advanced Publishing Program, Boston, 1980.

[2] M. E. Keskin and N. Ağiralioğlu, Simplified dynamic model for flood routing in rectangular channels, Journal of Hydrology 202(1-4) (1997), 302-314.

[3] M. A. Moghaddam and B. Firoozi, Development of dynamic flood wave routing in natural rivers through implicit numerical method, American Journal of Scientific Research 14 (2011), 6-17.

[4] S. Mungkasi, I. Magdalena, S. R. Pudjaprasetya, L. H. Wiryanto and S. G. Roberts, A staggered method for the shallow water equations involving varying channel width and topography, International Journal for Multiscale Computational Engineering 16(3) (2018), 231-244.

[5] G. S. Stelling and S. P. A. Duinmeijer, A staggered conservative scheme for every Froude number in rapidly varied shallow water flows, International Journal for Numerical Methods in Fluids 43(12) (2003), 1329-1354.

[6] B. A. Sulistyono and L. H. Wiryanto, Investigation of flood routing by a dynamic wave model in trapezoidal channels, AIP Conference Proceedings 1867 (2017), 020020.

[7] B. A. Sulistyono and L. H. Wiryanto, An analysis of the influence of variability rainfall on flow rate based on the watershed characteristics, IOP Conference Series: Earth and Environmental Science 124 (2018), 012001.

[8] L. H. Wiryanto and A. Achirul, An implicit finite difference method for a forced KdV equation, Jurnal Matematika 11 (2008), 1-5.

[9] J. Balbás and S. Karni, A central scheme for shallow water flows along channels with irregular geometry, ESAIM: Mathematical Modelling and Numerical Analysis 43(2) (2009), 333-351.

[10] X. Q. Zhang and W. M. Bao, Modified Saint-Venant equations for flow simulation in tidal rivers, Water Science and Engineering 5(1) (2012), 34-45.

[11] Y. Zhang, Simulation of open channel network flows using finite element approach, Communications in Nonlinear Science and Numerical Simulation 10(5) (2005), 467-478.

Bambang Agus Sulistyono paper 2

ORIGINALITY REPORT

2%	0%	2%	0%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	M P Hadi, N J L Syamsiyyah, L Annisa. "Unit hydrograph method for curve number validation in hydrological modeling: case study of Welaran Watershed, Karangsambung, Kebumen", IOP Conference Series: Earth and Environmental Science, 2020 Publication	2%
2	International Journal of Numerical Methods for Heat & Fluid Flow, Volume 23, Issue 1 (2013-01-05) Publication	<1%

Exclude quotes	Off	Exclude matches	Off
Exclude bibliography	On		

Bambang Agus Sulistyono paper 2

GRADEMARK REPORT

FINAL GRADE

/0

GENERAL COMMENTS

Instructor

PAGE 1

PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7

PAGE 8

PAGE 9